REVIEW

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On the documents submitted for a participation in the competition for an occupation of the academic position "Professor" at the Faculty of Economics in the Technical University - Gabrovo on **Research area: 4.** Natural sciences, mathematics and informatics, **Professional field 4.5.** Mathematics (Computational Mathematics), announced in the State Gazette no. 48 of 13. 06. 2025 and on the web site of the Technical University - Gabrovo, where Milena Radoslavova Racheva, Doctor of sciences (DrSci), an Assoc. Prof. from the Technical University - Gabrovo participates as a candidate – **the only one** candidate.

By Order № РД 3-01-387/19.09.2025 of the Rector of Technical University - Gabrovo, I was appointed as a member of the Scientific Jury of the competition for the occupation of the academic position "Professor" at the Technical University - Gabrovo on Research area: 4. Natural sciences, mathematics and informatics, Professional field 4.5. Mathematics (Computational Mathematics).

As a member of the jury I, obliged to write a review, have received all the necessary documents attached to the application of Associate Prof. Milena Radoslavova Racheva, DrSci to the Rector of the Technical University - Gabrovo for participation in the competition. The documents are well designed and arranged.

For the participation in the announced competition just one candidate (Associate Prof. Milena Radoslavova Racheva, DrSci) has submitted documents. She has enclosed necessary reports to satisfy the minimum national requirements and the additional requirements of the Technical University - Gabrovo.

Assoc. Prof. Prof. Milena Racheva, DrSci graduated a higher education in 1990, with a five-year period of study, which is equivalent to a master's degree, she acquired a PhD degree in 2003, won a competition for the academic position of associate professor in 2006, and successfully defended a DrSci thesis in 2014.

The candidate has an educational and scientific degree of doctor (PhD), thus satisfying the minimum national requirements for indicator "A" with 50 points. The PhD thesis is registered in NACID with scientific-metric indicators and satisfies the requirements for publications on it to have no less than 30 points.

Milena Racheva has been successively assistant professor 1992-2002, senior assistant professor 2002-2006 and associate professor from 2006 to the present. She was a visiting scientist at Chalmers Tekniska Högskola – Gothenburg, Kingdom of Sweden from 2004 to 2005.

The candidate presents 4 publications for participation in the competition with a total number of points 120 for group of indicators “В”. Assoc. Prof. Milena Racheva, DrSci applies to the competition with a total of 423 points for group of indicators “Г” and with 96 citations, which give 752 points for group of indicators “Д”. Two textbooks 53.33 (a single author in one of them and three authors for the second one). Both textbooks are indexed in the library data base COBISS. She is are Supervisor of one PhD student (a co-supervisor, but with a colleague from different research area) and thus she gets 50 points. A participation in 4 national scientific projects, gives her additional 40 points. She has a scientific degree “Doctor of Sciences” so it means 75 points to be added. Finally, Assoc. Prof. Milena Racheva, DrSci presents 213.33 points for group of indicators “Е”.

This quick review shows that all minimum national requirements are met by the candidate.

The additional requirements of the Technical University – Gabrovo are satisfied as follows: Minimum 30, of which at least 5 as a single author and 3 with IF (WoS) – are satisfied with 37 scientific publications, 5 that are with a single author and 3 with an IF (WoS); minims 20 citations, author or co-author of 2 textbooks, supervisor or co-supervisor of one PhD student, all these were satisfied by the comments in the previous paragraph; project or contract management of minimum 3 ones is satisfied.

This quick review shows the additional requirements of the Technical University – Gabrovo are fulfilled by the candidate

GENERAL CHARACTERISTICS OF THE CANDIDATE'S ACTIVITY

ASSESSMENT OF THE TEACHING AND PEDAGOGICAL ACTIVITY

I do not know the candidate personally; I have not even had the opportunity to attend her conference presentation. This can sometimes be considered a positive thing, for the jury to have members who will evaluate the candidate only on the basis of the presented scientific output.

EVALUATION OF SCIENTIFIC AND SCIENTIFIC-APPLIED ACTIVITIES

The candidate has structured her scientific contributions correctly into three main fields.

The main contributions of the candidate ate in the first field. They are continuation of her investigations for PhD, DrSci degrees, and Assoc. Prof. position, in the field of approximations of differential equations. The main problem is the eigen value problem – the approximations of the eigenvalues and finding the eigen functions.

The eigenvalue problem plays a fundamental role in solving differential equations of any order, including both ordinary and partial types. It originates in the 18^{th} century in the works of d’Alembert, Euler, Bernoulli dealing with the equation of the vibrating strings. Throughout the year the eigenvalue problem has been generalized, altered, and enriched with different techniques.

The main contributions of the candidate are in the field of the eigenvalue problem [8.1-8.11, 8.13-8.14,8.16-8.27], which are 25 articles out of 37 (68%). Development of special basis functions

(integral-type, nonconforming), postprocessing algorithms, and coupling procedures for two-sided eigenvalue bounds.

The candidate has used a wide range of methods and techniques to solve the eigenvalue problem. The techniques and methods have been carefully selected to achieve the desired results. The eigenvalue problem is well known to be unsolvable in the general case, so it is necessary to restrict the class of functions for which the solution is sought or to find eigenvalue estimates that are good enough to investigate the model from which the differential equation or system arose.

METHODS AND TECHNIQUES

I would like to mention three basic methods, applied in the presented. These are the finite element method, the mixed variational formulation, and the reduction of differential equations to systems of lower order differential equations and be linear of the spectral parameter is in a linear form in the equation. This kind of decreasing the order of the differential equation and getting a one with a linear parameter have its cost by increasing the number of functions and equations. This is not a draw back because the new system of differential equations can be handled easier with numerical methods.

I will say a few words about the basic techniques, that are used. The finite element method (FEM) is a well-known one for numerically solving differential equations. It originates from engineering problems, and is widely used nowadays in all fields of applied mathematics, where the models generate differential equations or systems. FEM is a general numerical method for solving partial differential equations in two- or three-space variables (i.e., some boundary value problems). FEM has been developed significantly covering wide classes of problems. The presented works by the candidate involve numerical solutions of higher order differential equations. The method approximates the unknown functions over the domain. The simple equations that model these finite elements are then assembled into a larger system of equations that models the entire problem. The introduction of the method can be traced back to the early 1940s. It was also independently rediscovered in China by Feng Kang in the late 1950s and early 1960s, based on the computations of dam constructions, where it was called the “finite difference method” based on variation principles. Although the approaches used by these pioneers are different, they share one essential characteristic: the mesh discretization of a continuous domain into a set of discrete sub-domains, usually called elements. In numerical analysis, a mixed finite element method, is a variant of the finite element method in which extra fields to be solved are introduced during the posing a partial differential equation problem. A mixed variational formulation is where multiple quantities are treated as independent variables within a single variational principle or equation. The works presented for the contest involve the mentioned above methods, used cleverly to get the desired results. New ideas are presented, which widens the known methods and increases the classes of equations that can be investigated.

New ideas are presented that expand the scope of known methods and increase the classes of differential equations that can be studied using them.

EVALUATION OF SCIENTIFIC AND SCIENTIFIC-APPLIED ACTIVITIES

Here, I will briefly outline the results obtained in the papers submitted for the competition.

In the studies [8.1] and [8.2], an algorithm for the linearization of quadratic Sturm–Liouville spectral problems for differential equations of the second and fourth order has been developed and analyzed. In [8.1] and [8.2], a novel approach is proposed, where through a special procedure a system of differential equations is obtained in which the spectral parameter λ appears only in a linear form. A theorem has been proved providing an error estimate for the eigenvalues of the quadratic Sturm–Liouville problem [8.2]. For problems of the fourth order, several linearization schemes have been proposed, including the combination of linearization with order reduction [8.3]. A significant contribution of the works [8.4] and [8.14] lies in demonstrating that, for model spectral problems of the sixth order, the proposed algorithm allows the use even of linear finite elements within the mixed finite element framework. A theorem for the ½ order error estimate of the eigen values when approximated with linear finite elements has been established. These papers also discuss matrix representations and computational aspects related to the implementation of the considered problems.

The publications [8.1–8.4] and [8.14] follow a common computational approach aimed at achieving two essential objectives: first, the linearization of quadratic Sturm–Liouville problems, and second, the reduction of the degree of the polynomial spaces used for the approximation of the spectra of even-order differential operators. Among the applications discussed, special attention is given to problems defined over unbounded domains, which frequently arise in engineering practice. In this context, a combined approach has been proposed: the bounded part of the domain is discretized using finite elements (FEs), while the exterior part is covered with infinite large elements (ILEs) [8.5]. The main achievement illustrated in these works is the construction of a conformal coupling between the two types of elements, ensuring the boundedness of the solution. A specific system of basis functions has been developed, which provides a conformal FE/ILE method. This result has inspired other similar coupling techniques for model problems posed in unbounded regions.

Fractional-order integral and differential operators are studied. This type of operators provides an effective tool for describing phenomena with “memory” in various materials. The basic equations of the dynamic viscoelastic problem are of the integro-differential hyperbolic type. In [8.6], a complex case of an integro-differential equation describing fractional-order dynamic viscoelasticity is analyzed, in which the convolution kernel is weakly singular. The solution is based on the discontinuous Galerkin method (Discontinuous Galerkin method). The numerous citations to [8.6] confirm the scientific weight and applicability of the obtained results.

In the field of numerical analysis, the so-called two-sided estimates of the eigenvalues are particularly valuable - i.e. obtaining both an upper and a lower bound simultaneously. In the papers [8.7] and [8.20], an original algorithm for deriving two-sided estimates is presented, without having to solve the spectral problem again. The proposed a posteriori procedure provides a significant increase in the accuracy of the numerical results. The authors prove theorems for two-sided estimates for second- and fourth-order problems and analyze computational aspects related to the application of the algorithm.

In the numerical experiments, nonconforming finite elements are initially used to obtain lower bounds for the eigenvalues. Specifically, Crouzeix–Raviart elements are applied for second-order problems [8.7], while Adini elements are employed for fourth-order plate vibration problems [8.20]. The deliberate relaxation of continuity requirements for the approximating (piecewise-polynomial) function spaces within FEM brings several advantages, such as simplified approximation of the divergence-free condition in the Stokes and Navier–Stokes equations, the generation of sparser matrices, and the possibility of deriving lower eigenvalue estimates.

In [8.8], a rectangular version of the Morley element is studied together with computational features relevant to fourth-order problems, while [8.18] analyzes in detail the three possible types of basis polynomials and the relationships among them for the rectangular Morley element. It is well known, following the min–max principle, that conforming FEM solutions always approximate eigenvalues from above. The above-mentioned studies, however, propose algorithms for constructing two-sided estimates based on a single finite-element computation, eliminating the need for any additional solution of the spectral problem.

Particularly original is the author's idea of employing an interpolational approach, in which the a posteriori estimates are derived through an appropriate interpolant of the already obtained finite element approximation. In practice, for second-order problems, it is both convenient and efficient—when using a triangular discretization—to construct interpolants associated with the nonconforming Crouzeix–Raviart (C–R) element and its extended version (EC–R) [8.11], [8.13], [8.25]. For quadrilateral finite element discretizations, similar procedures are applied [8.9], [8.26].

The author introduces a substantial computational concept — the replacement of pointwise degrees of freedom with integral ones — which simplifies implementation and improves numerical stability [8.25], [8.26]. The method is applicable to both second- and fourth-order problems. In [8.11], a seven-point finite element with integral degrees of freedom is developed, suitable for obtaining two-sided eigenvalue bounds and for building nonconforming interpolants in a simpler and more accessible form. It is also highly effective for problems defined on overlapping subdomains.

Another innovative idea consists in aggregating groups of finite elements into macroelements, enabling both local and global acceleration of convergence. This approach leads to ultraconvergence effects, whereby the order of the mesh-dependent error increases by at least two. The so-called patch-recovery techniques, though theoretically demanding, prove to be efficient and easily applicable in practice. Paper [8.10] presents a comprehensive investigation of boundary-value problems of the fourth order involving the biharmonic operator, provides proofs of ultraconvergence results, and details numerical algorithms for implementing the proposed a posteriori procedure.

The papers [8.16] and [8.17] examine the solution of algebraic systems obtained after applying the isoparametric finite element method (FEM). The purpose of this approach is to achieve a more accurate and convenient numerical approximation of boundary-value problems when the computational domain has curvilinear or piecewise-curvilinear boundaries. These two studies are closely related and analyze the qualitative characteristics of large linear systems arising from the

implementation of the isoparametric FEM. The convergence of a two-level hierarchical condensation method is proved, leading to an upper bound estimate for the constant γ in the classical strengthened Cauchy inequality. The results are illustrated for second- and fourth-order boundary-value problems, and quantitative assessments of the effectiveness of the proposed procedures are presented.

In [8.22], a model of a pneumatic cylinder system is studied, representing the interaction between a pneumatic/hydraulic valve and an actuator. The objective is to describe the transient processes in the dynamics of fluid-power systems. By applying the Laplace transform, a relationship between the cylinder length and the time delay is derived.

Paper [8.23] presents a new numerical method for determining the eigenvalues of a real 3×3 matrix, based on finding the positive root of a specific cubic equation involving the trace and determinant of the matrix. This root enables the direct computation of all three eigenvalues within a precisely isolated interval. In [8.24], a mathematical model of a rotating cylindrical body is examined, and its stability is analyzed under varying axial forces and torques. The study employs a mixed variational approach for approximating the fundamental eigenpair and introduces two mixed variational models with symmetric forms, suitable for numerical implementation. In [8.27], a mathematical model of bending vibrations of a deformable elastic rod is considered, with multiple boundary conditions investigated—most of them motivated by practical applications. Theoretical results address the loss of symmetry and the occurrence of eigenvalues within the boundary conditions themselves.

Publications [8.35–8.37] focus on the applications of trigonometric polynomials and Fourier transforms in the analysis of periodic functions and signals. The approximation of periodic functions by harmonics of a given order and the derivation of the corresponding frequency spectra are central to both signal theory and electrical engineering. In [8.35], convolution transformations of commonly used signals are obtained, and through the inverse Fourier transform (FT) smooth original functions are reconstructed. A convolution-based inverse FT method is presented, allowing the computation of improper integrals of positive integer powers of the sinc-function. In [8.36], the concept of discrete approximation of Fourier harmonics is developed for cases where the periodic function is partially or entirely unknown. The author proposes interpolational and metric approaches for computing harmonic coefficients, implemented using the computational capabilities of Wolfram Cloud. Publication [8.37] extends this framework to both direct and inverse problems for periodic functions, emphasizing the role of the generalized Dirac delta function and the numerical approximations of Fourier coefficients. Across these works, the author advocates that in engineering practice, where data are predominantly derived from measurements, computational mathematics plays a leading role. Accordingly, new approximation techniques are introduced, integrated with modern mathematical software for their realization.

Two papers further discuss the practical and pedagogical aspects of employing mathematical software in research and education. It is emphasized that such software has become a fundamental tool in scientific investigation, teaching, and engineering applications. In [8.30], a comparative analysis of leading mathematical software systems available on the market is performed, offering pragmatic guidelines for their effective use. Particular emphasis is placed on Wolfram Cloud and

Wolfram Alpha, whose potential has been validated through successful teaching practice. Paper [8.31] focuses on the application of mathematical software in distance learning, addressing both preparation and assessment. Real-world teaching scenarios are analyzed, discussing challenges and advantages of the remote format. The author's writing style in these works is intentionally stimulating and discussion-oriented, aiming to provoke reflection on the teaching of mathematical disciplines in technical universities.

A group of publications: [8.12], [8.15], [8.32-8.34] and related works—focuses on scientific and applied research in energy efficiency and renewable energy sources. The thematic direction is consistent with the national and EU priorities for improving energy efficiency, environmental sustainability, and quality of life. In [8.12], the electrical subsystem of a small pumping station is analyzed. Based on computational modeling, the electrical losses in the driving motors are evaluated, and modernization measures are proposed for enhancing the efficiency of the water-supply system through the integration of new electric-drive equipment. Study [8.32] examines a low-power photovoltaic system implemented via a flexible module as a modern alternative energy source, describing the technical characteristics of its components, digital measurement techniques, and experimental findings. Publications [8.15] and [8.33] address the use of hydropower, analyzing the operation of electrical subsystems in small hydroelectric plants. Mathematical processing of operational data and statistical analyses are performed, leading to summarized conclusions regarding their performance indicators.

In [8.28], [8.29] and [8.34], the author investigates contemporary aspects of electric mobility. Paper [8.28] provides a global overview of the production, sales, and utilization of electric vehicles, while [8.29] analyzes the status of the sector in Bulgaria during the respective period. A key contribution is the development of a methodological framework for the study and evaluation of parameters and socio-economic factors affecting the evolution of electromobility, as well as the formulation of policy and infrastructure recommendations. Publication [8.34], a 34-page plenary report delivered at the annual Energy Forum under the auspices of the Scientific and Technical Union of Power Engineers in Bulgaria, presents a systematized analysis of the market for new and used electric vehicles. It includes detailed technical and economic characteristics, trends, and recommendations. This work represents a valuable contribution to understanding the challenges and potential of electric mobility in the context of sustainable development.

I also think it is good practice for candidates for academic growth to also submit publications in journals and conferences not indexed in the WoS and SCOPUS databases, because these conferences and journals must also collect citation data for entry into the mentioned databases. I am impressed by the candidate's regular participation in the International Conference UNITECH – Gabrovo and publications in Mathematical Modeling, a newly emerging journal.

COMMENT ON THE TWO PRESENTED TEXTBOOKS

The textbook "Applied Mathematics" by Assoc. Prof. Milena Racheva, D.Sc., is intended for students in engineering specialties. The work is characterized by a clear and logically consistent structure. The author systematically presents the main topics related to data approximation, Fourier transformations, and elements of probability theory and statistics, ensuring coherence and smooth

transitions between the sections. Particularly valuable is the practical approach, through which abstract mathematical concepts are connected to concrete engineering applications. The exposition is clear, precise, and accessible, while maintaining academic rigor. The numerous examples and illustrations support comprehension and mastery of the material, and the use of mathematical software (WolframAlpha, Wolfram Cloud) demonstrates modern thinking and orientation toward the digital age. The textbook includes a rich set of methodological guidelines, which facilitate both student learning and teaching practice.

The textbook "Modern Mathematical Methods for Engineers" represents an in-depth and methodically well-structured work devoted to the application of mathematical models and numerical methods in engineering sciences. The authors successfully combine the theoretical foundations of mathematical modeling with practical engineering applications, thus building a bridge between fundamental science and engineering practice. The content is clearly organized into three main parts — mathematical modeling, basic theory of computational methods, and the finite element method — providing logical consistency and a smooth transition from theory to practice. The exposition is distinguished by precision, terminological accuracy, and pedagogical clarity. The examples from electrical engineering and physics demonstrate the practical applicability of the discussed methods and encourage independent analytical thinking. Of particular value is the emphasis on variational methods and the finite element method, which are among the most advanced tools for solving engineering problems. The textbook is written at a high academic level, yet in an accessible and logically coherent style.

In conclusion, the applied aspects of mathematics presented in both textbooks make them suitable not only for undergraduate engineering students, but also for master's and doctoral students specializing in applied mathematics.

CONCLUDING NOTES ON PUBLICATION ACTIVITY

I believe that it is good practice for candidates for academic growth to submit publications not only in journals and conferences indexed in the WoS and Scopus databases, but also in those that are not yet indexed, since these journals also collect citations and fundamental results, information necessary for inclusion in the aforementioned databases.

I am impressed by the candidate's regular participation in the International Conference UNITECH — Gabrovo, as well as by her publications in the journal Mathematical Modeling — a new and developing scientific journal.

I have not found "plagiarism" in the works of the candidate in the sense of the "Law on the Development of the Academic Staff in the Republic of Bulgaria" in the Republic of Bulgaria.

CRITICAL NOTES

I recommend that the candidate start working with doctoral students in the field of mathematics. In the presented works, the illustrative examples are mainly with equations, where the exact values are first found, and then the numerical series that converge to them are constructed. This is a well-known practice in the subject considered in the review, which is used to illustrate the presence of

convergence. I recommend that, in addition to this type of illustrations, which represent a well-known approach, examples should also be included where the exact values cannot be found. This will illustrate the applicability of the proposed techniques to tasks of higher difficulty. I suggest that the research be deepened in the direction of the stability of approximated eigenvalues. In my opinion, it would be very valuable to publish a monograph summarizing the ideas developed - it would be useful for young researchers who are starting work in this scientific field. I also recommend that the applicant adhere to the widespread international practice of arranging authors alphabetically, regardless of their contribution, as sometimes a single detail can be decisive for the acceptance or rejection of a publication..

CONCLUSION

In my opinion the candidate Associate Prof. Milena Radoslavova Racheva, DrSci has obtained enough results both in quality and quantity. The presented documents meet the requirements, conditions and criteria of the Law on the Development of the Academic Staff in the Republic of Bulgaria, Rules for applying of the mentioned above law, Rules for the conditions and order for acquiring academic degrees and academic positions at Technical University - Gabrovo to occupy the academic position “Professor”. Therefore, I give my **strictly positive assessment and I recommend to the Scientific Jury to prepare a report-proposal to the Scientific Council of the Faculty of Economics at the Technical University - Gabrovo for the election of Assoc. Prof. Milena Radoslavova Racheva, DrSci for the academic position “Professor”** in the Technical University - Gabrovo in Research area: 4. Natural sciences, mathematics and informatics, Professional field 4.5. Mathematics (Computational Mathematics)

02.11.2025 Signature: .....
Plovdiv /Prof. Boyan Zlatanov, Dr.Sci./